Q1 - first order system ✓

Q2 - Second order system ✓

Q3 - Error Type number Final value ✓

Q4 - Block Diagram Reduction ✓

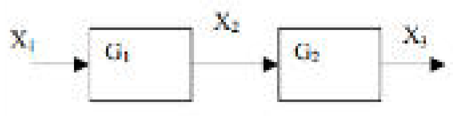
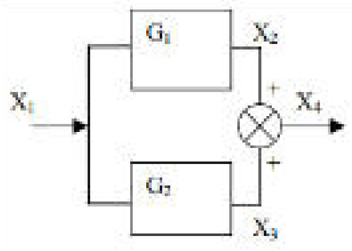
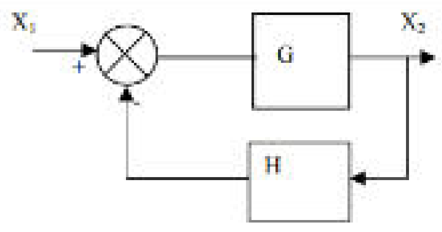
Q5 - Control Strategies ✓

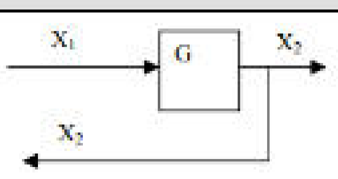
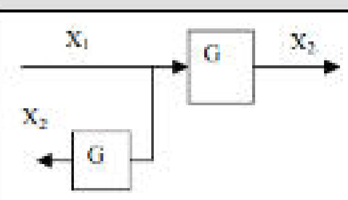
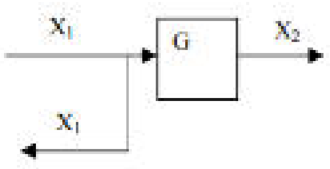
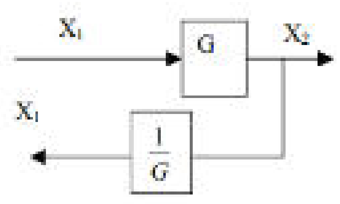
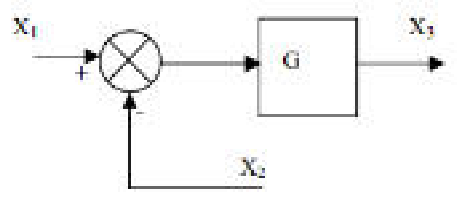
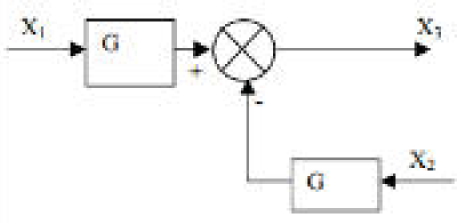
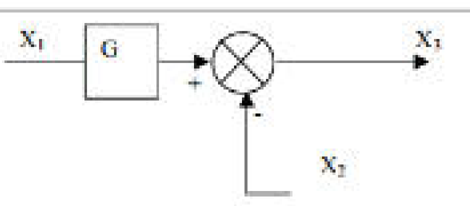
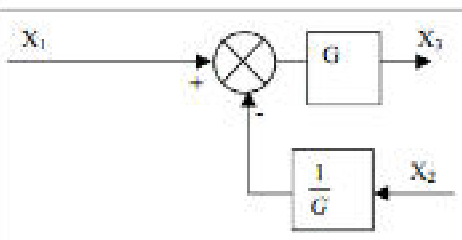
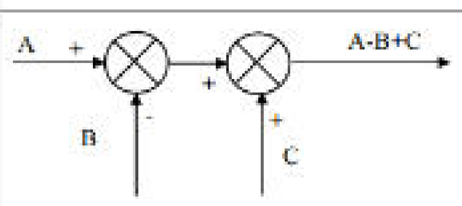
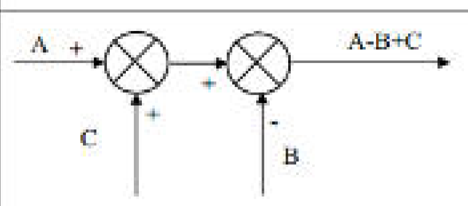
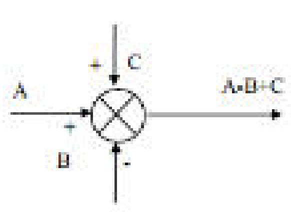
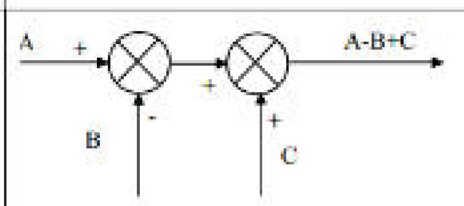
Q6 - instructment ✗

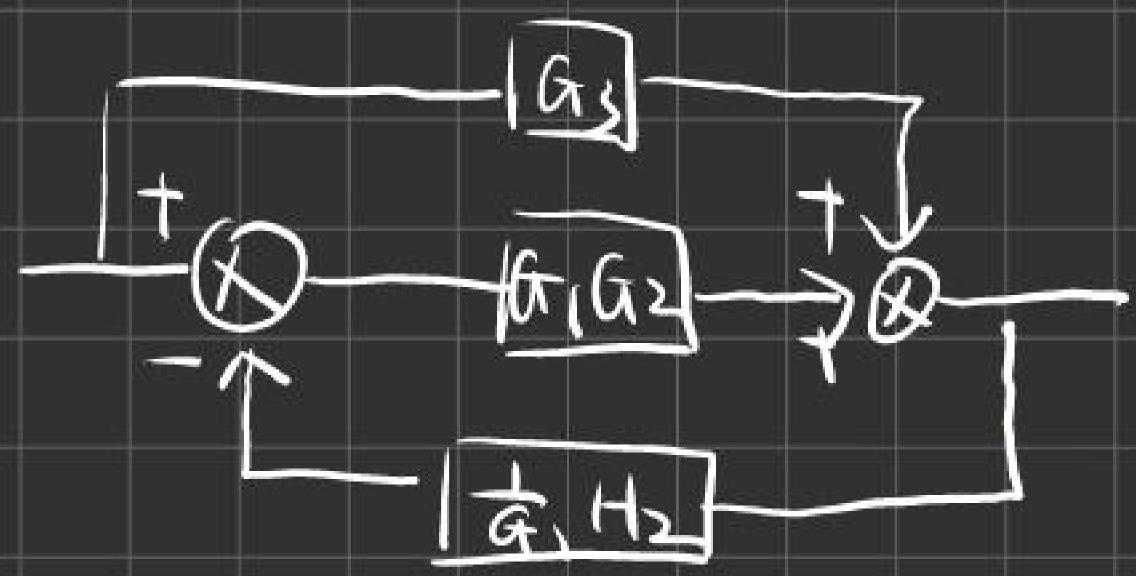
Any four of six

Block diagram reduction ✓

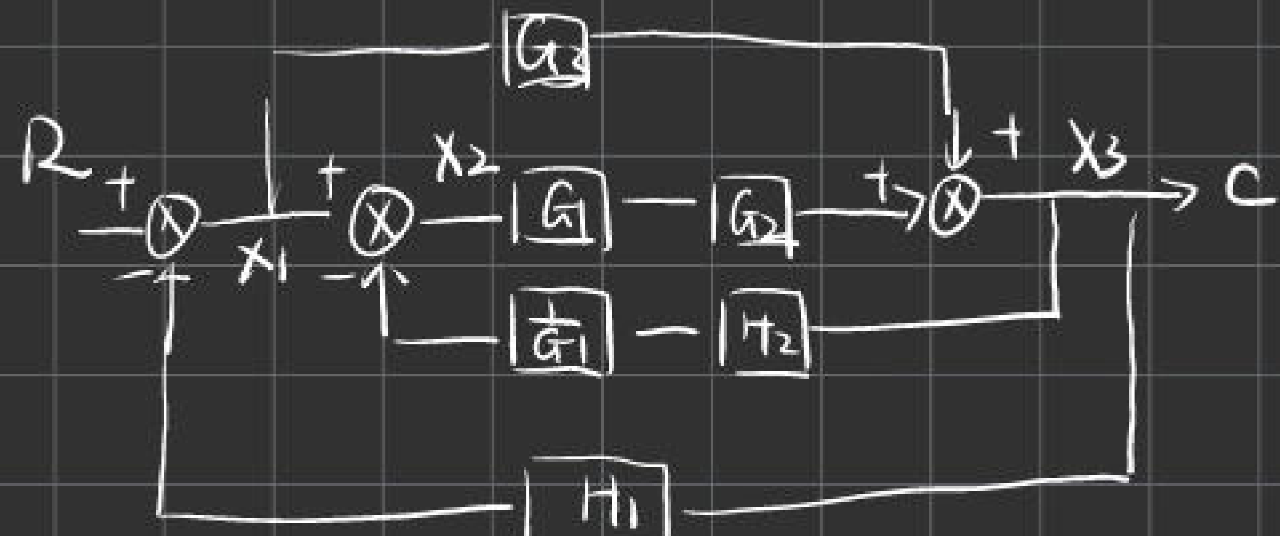
The 3 ways of reducing a block diagram.

Transformation	Original diagram	Equivalent diagram
Blocks in series $X_2=G_1.X_1$ $X_3=G_2.X_2 = G_2(G_1.X_1)$ $=(G_1.G_2)X_1$		
Blocks in parallel $X_2=G_1.X_1$ $X_3=G_2.X_1$ $X_4=X_2+X_3=G_1.X_1+G_2.X_1$ $=(G_1+G_2)X_1$		
Eliminating a feedback loop Negative feedback $\frac{G}{1+GH}$ Positive feedback $\frac{G}{1-GH}$		

Transformation	Original diagram	Equivalent diagram
Moving a pickup point ahead of a block $X_2=G.X_1$		
Moving a pickup point behind a block $X_2=G.X_1$ $\frac{1}{G}.X_2=\frac{1}{G}.G.X_1=X_1$		
Moving a summing point behind a block $X_3=G(X_1-X_2)=G.X_1-G.X_2$		
Moving a summing point ahead of a block $X_3=G.X_1-X_2$ $=G(X_1-\frac{1}{G}.X_2)$		
Moving Signals on Summing Blocks		
Splitting Signals in Summing Blocks		



$$\frac{G_1G_2+G_3}{1+H_2G_2}$$



$$\begin{cases} X_3 = X_1G_3 + X_2G_1G_2 \\ X_2 = X_1 - X_3\times H_2\times \frac{1}{G_1} \end{cases}$$

$$\Rightarrow X_3 = X_1G_3 + G_1G_2X_1 - X_3\times H_2\times G_2$$

$$\Rightarrow X_3 = \boxed{\frac{(G_1G_2+G_3)X_1}{1+H_2G_2}}$$

$$\Rightarrow \frac{\frac{G_1G_2+G_3}{1+H_2G_2}}{1+\frac{G_1G_2+G_3}{1+H_2G_2}\times H_1}$$

Zero order system (perfect system)

where the output is directly proportional to the input

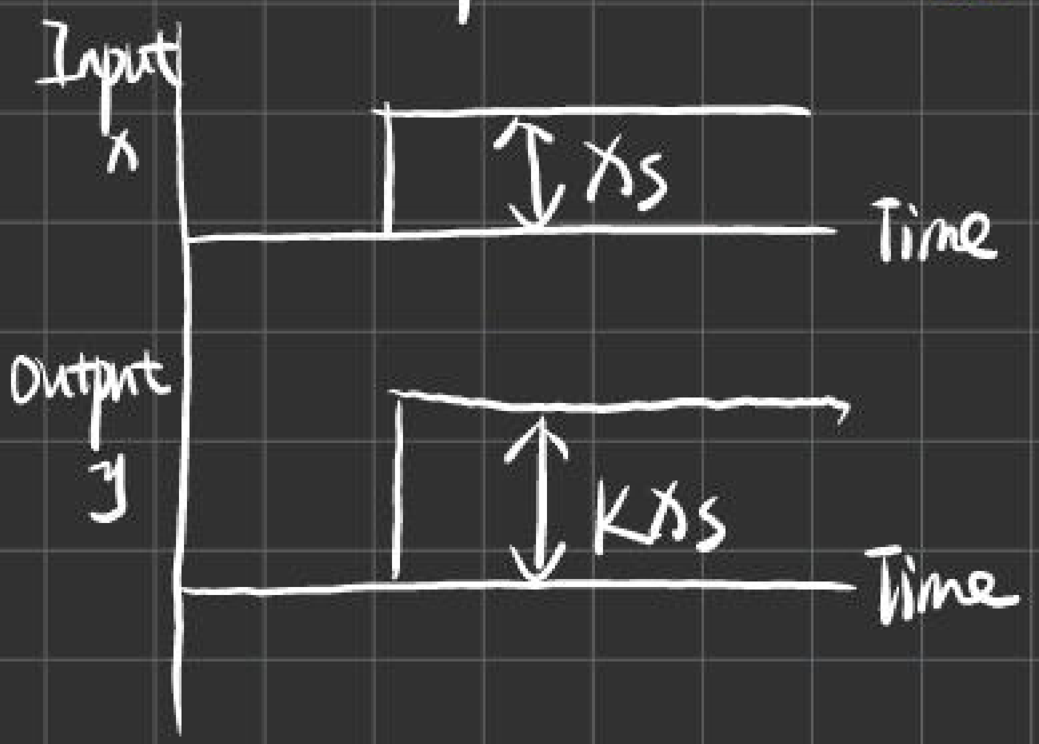
Equation: $a_0 y = b_0 x \Rightarrow y = \frac{b_0}{a_0} x = kx$ k: static sensitivity = Gain 增益

若输入变为正弦波

$x = A\sin\omega t$

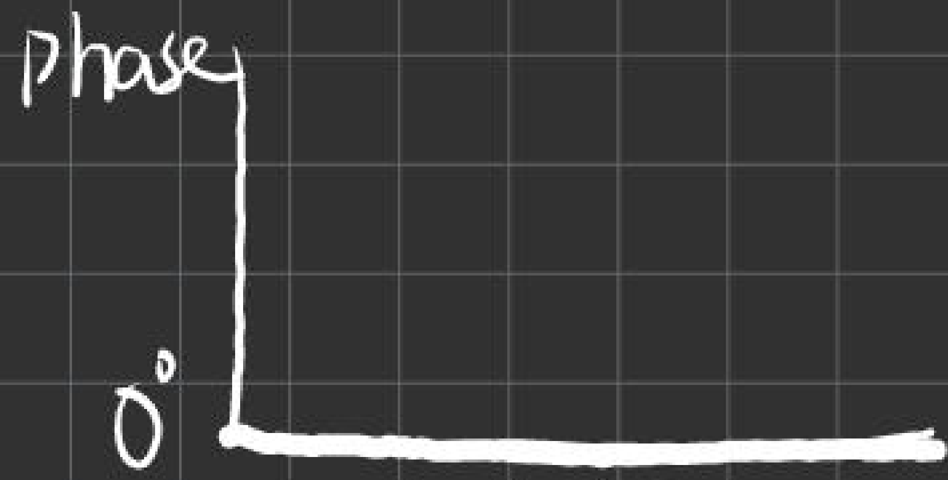
则 $y = kx = kA\sin\omega t$

Time response: 时域响应



Frequency response: 频域响应

幅频图



相频图

First order system (一阶系统)

形式: $a_1 \frac{dy}{dt} + a_0 y = b_0 x \Rightarrow \frac{a_1}{a_0}\frac{dy}{dt} + y = \frac{b_0}{a_0} x$

$\Rightarrow \tau \frac{dy}{dt} + y = K x$

τ: time constant 时间常数

K: Static Sensitivity (Gain)

$\tau \times$ 输出变化率 + 输出 = K × 输入

当 output settle down,

则 rate of change of output = 0

则 输出 = K × 输入

K × input.

τ的单位是秒 (s)

K的单位是 $\frac{y}{x}$

$\frac{y\text{的量纲}}{x\text{的量纲}}$

当 $t=\tau$ 时 $y = 0.632K \cdot x$

when $t=\tau$, the value of output is 63.2% of the final steady value.

Laplace Transform

$$F(s) = L[f(t)] = \int_0^\infty f(t)e^{-st}\,dt$$

常用 $\frac{dy}{dx} \to sY(s)$ $\frac{d^2y}{dx^2} \to s^2Y(s)$

对于 first order system ; $\tau\frac{dy}{dx} + y = kx$

拉氏变换 ; $\tau sY(s) + Y(s) = kX(s)$

则 transfer function : $G(s) = \frac{Y(s)}{X(s)} = \frac{K}{1+\tau s}$

对于阶跃输入 ; $X(s) = \frac{1}{s}$ 则 $Y(s) = \frac{K}{1+\tau s} \times \frac{1}{s} = \frac{k}{s} + \frac{-k\tau}{1+\tau s}$

注意 : $\frac{1}{s} \sim 1$, $\frac{1}{s^2} \sim t$, $\frac{1}{s+\frac{1}{\tau}} \sim e^{-t/\tau}$ $= k\left(\frac{1}{s} - \frac{\tau}{1+\tau s}\right)$

$= k\left(\frac{1}{s} - \frac{1}{\frac{1}{\tau}+s}\right)$

则 $y = k \times (1 - e^{-t/\tau})$

Second order system

$$a_2\frac{d^2y}{dt^2} + a_1\frac{dy}{dt} + a_0 y = b_0 x$$

二阶系统标准的传递方程 : $G(s) = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}$

w_n : nature frequency

ξ : damping ratio

w_n ; nature frequency (rad/s) frequency at which the system would oscillate at if there was no damping

a measure of how well a system can respond to fast changing inputs

High $w_n \Rightarrow$ respond quickly low $w_n \to$ slow respend

ξ ; Damping ratio. represent friction in the system.

control the size of oscillations and how quick they decrease or die out.

$\xi > 1$, two negative real and unequal roots
over damped response

$\xi < 1$ under damped response

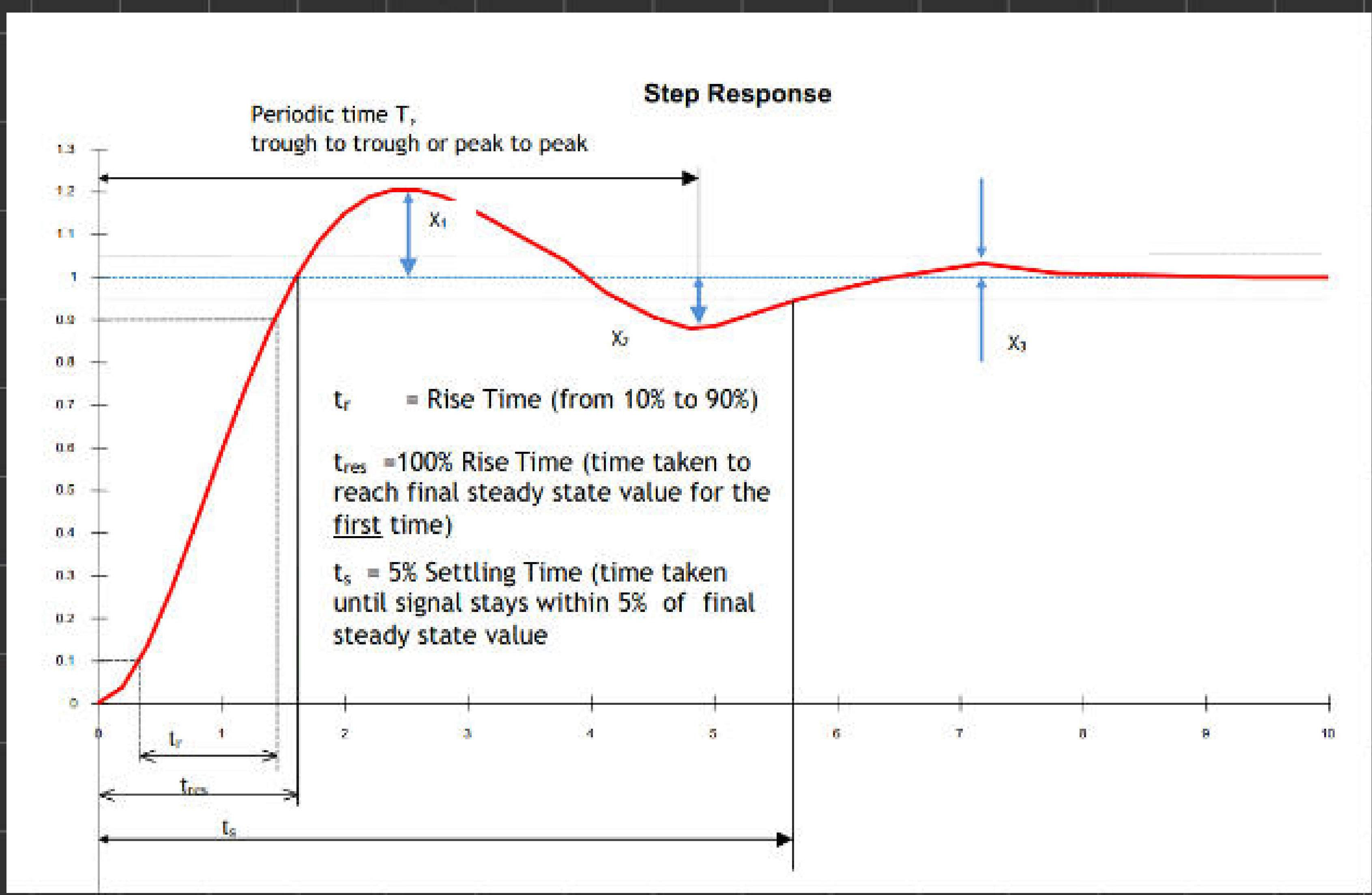
$\xi = 1$ critically damped system

Step Response Specification

response time : (t_{res}), time taken for the system output to rise from 0% to the first cover 100%

rise time : (t_r), time taken for the system output to rise from 10% to 90% of final steady value

settling time : (t_s), time taken for the system output to reach and remain within a certain percentage tolerance
2%. 5%



%overshoot: $e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \times 100\ \%$

阻尼频率: $w_d = \sqrt{1-\xi^2}\, w_n$

T_{res}(响应时间): $\frac{\pi-\varphi}{w_n\sqrt{1-\xi^2}}$ $\varphi = \arccos\xi$

2% settle time: $t_{s2\%} = \frac{4}{\xi w_n}$

5% $t_{s5\%} = \frac{3}{\xi w_n}$

rise time: 10%~95% 考试出来的

poles：极点（分母为0） zeros：零点（分子为0）

对于多项式传递函数 $G(s)=\frac{C(s)}{R(s)}=K\frac{(s-z_1)(s-z_2)\cdots}{(s-p_1)(s-p_2)\cdots}$

例：$G(s)=\frac{s+2}{s+5}$ 对于阶跃响应：$C(s)=\frac{1}{s}\cdot\frac{s+2}{s+5}$

$$C(s)=\frac{1}{s}\frac{s+2}{s+5}=\frac{K_1}{s}+\frac{K_2}{s+5} \Rightarrow K_1=2/5 \quad K_2=3/5$$

$$C(s)=\frac{2/5}{s}+\frac{3/5}{s+5}$$

$\downarrow$

$$c(t)=\underbrace{\frac{2}{5}}_{\text{forced response}}+\underbrace{\frac{3}{5}e^{-5t}}_{\text{natural response}}$$

Laplace Table

f(t)	F(s)
Unit Impulse $\delta(t)$	1
Unit Step 1 or size A	$\frac{1}{s}$ or $\frac{A}{s}$
Unit Ramp t or At	$\frac{1}{s^2}$ or $\frac{A}{s^2}$
t^2	$\frac{2}{s^3}$
$t^n(n=1,2,3,4,5.....)$	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	$sF(s)-f(0)$
$\frac{d^nf(t)}{dt^n}$	$s^nF(s)-s^{n-1}f(0)-....f^{n-1}(0)$ Where $f^n(0)=\left[\frac{d^nf(t)}{dt^n}\right]_{t=0}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
t^ne^{-at}	$\frac{n!}{(s+a)^{n+1}}$
$\frac{1}{(b-a)}\times\left(e^{-at}-e^{-bt}\right)$	$\frac{1}{(s+a)(s+b)}$
$\frac{1}{(b-a)}\times\left(be^{-bt}-ae^{-at}\right)$	$\frac{s}{(s+a)(s+b)}$
$\frac{1}{ab}\left[1+\frac{1}{(a-b)}\times\left(be^{-at}-ae^{-bt}\right)\right]$	$\frac{1}{s(s+a)(s+b)}$
Sin ωt	$\frac{\omega}{s^2+\omega^2}$
Cos ωt	$\frac{s}{s^2+\omega^2}$
$e^{-at}Sin\,\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}Cos\,\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_nt}Sin(\omega_nt\sqrt{1-\zeta^2})$ $\zeta<1$	$\frac{\omega_n^2}{s^2+2\zeta\omega_ns+\omega_n^2}$
$\frac{-1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_nt}Sin(\omega_nt\sqrt{1-\zeta^2}+\phi)$ Where $\phi=\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$	$\frac{s}{s^2+2\zeta\omega_ns+\omega_n^2}$
$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_nt}Sin(\omega_nt\sqrt{1-\zeta^2}+\phi)$ Where $\phi=\cos^{-1}\zeta,\zeta<1$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_ns+\omega_n^2)}$

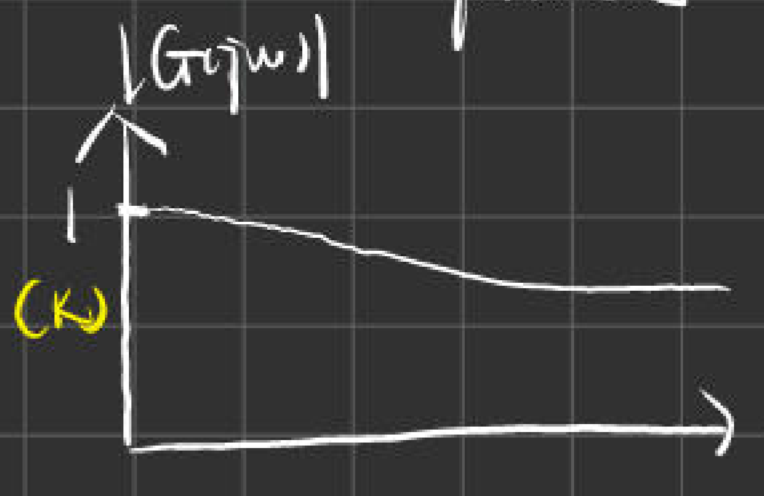
Frequency Response

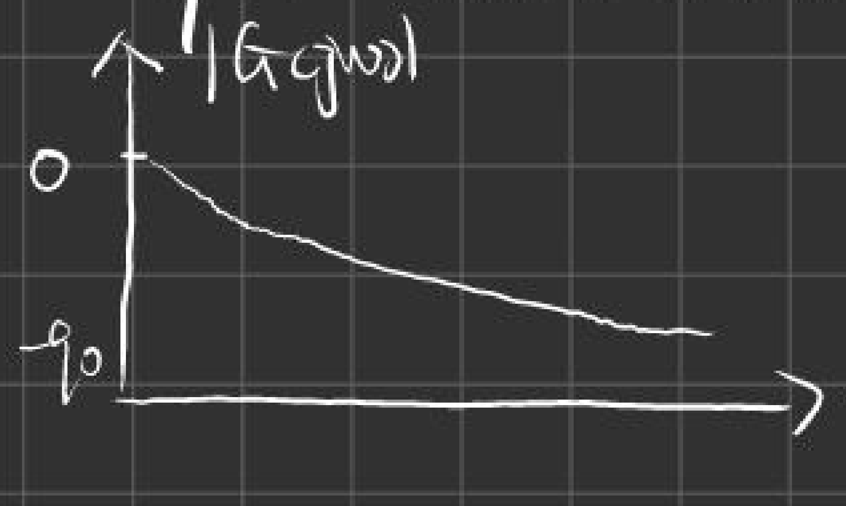
$G(s)=\frac{Y(s)}{X(s)}$ $\quad G(j\omega)=\left|\frac{Y}{X}\right|=\frac{K}{1+\omega\tau j}$

First order system:

amplitude ratio: $|G(j\omega)|=\left|\frac{Y}{X}\right|=\frac{K}{\sqrt{1+\omega^2\tau^2}}$ 幅频特性

phase angle: $\phi=-\arctan(\tau\omega)$

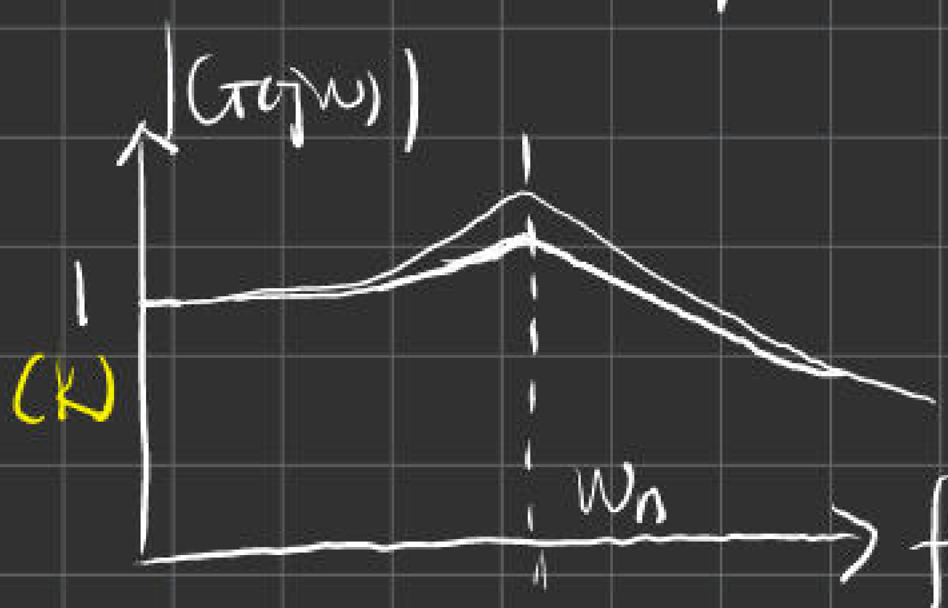




$G(j\omega)=|G(j\omega)|\,e^{j\varphi(\omega)}$

Second order system:

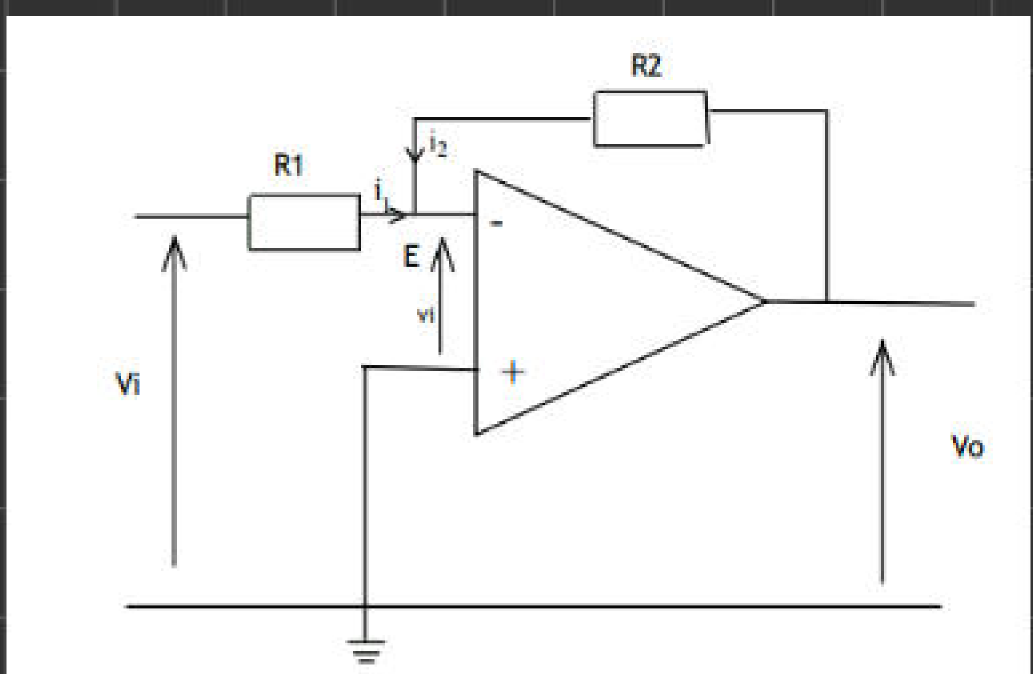
amplitude ratio: $|G(j\omega)|=\frac{K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$



Error: 由频率引起的幅值变化差值

$$\text{error}=\frac{|G(j\omega)|-K}{K}=\frac{\frac{1}{\sqrt{\ }}-1}{1}\times 100\%$$

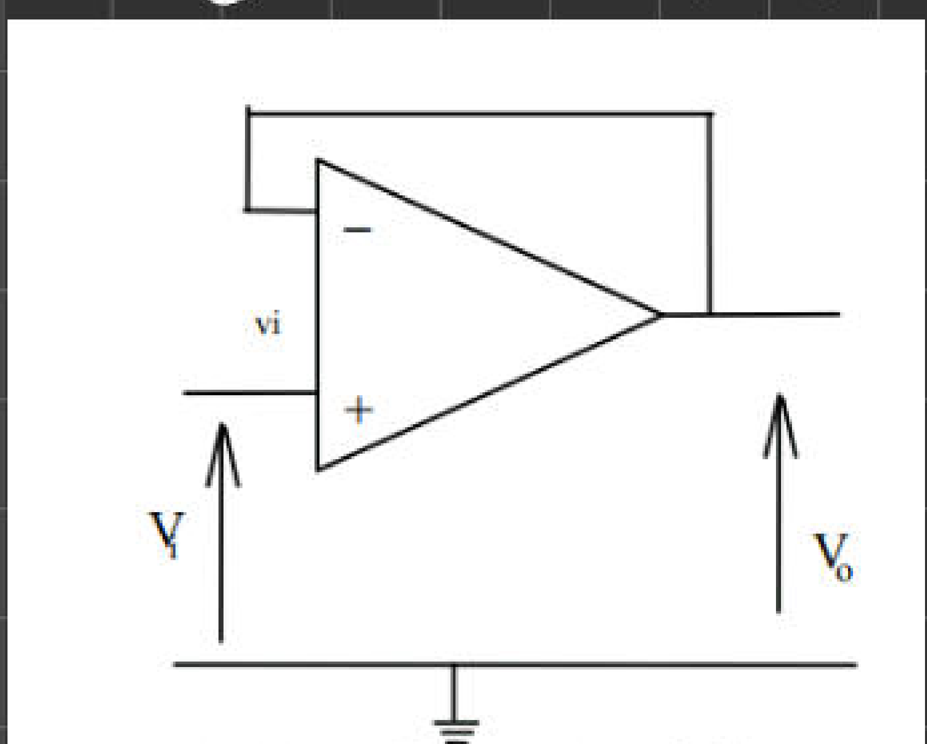
Inverting Amplifier



Inverting Amplifier

$$\left.\begin{aligned}i_1&=-i_2\\ i_1&=\frac{V_i}{R_1}\\ i_2&=\frac{V_o}{R_2}\end{aligned}\right|\quad \frac{V_i}{R_1}=-\frac{V_o}{R_2}\Rightarrow\frac{V_o}{V_i}=-\frac{R_2}{R_1}$$

Unity Gain Amplifier

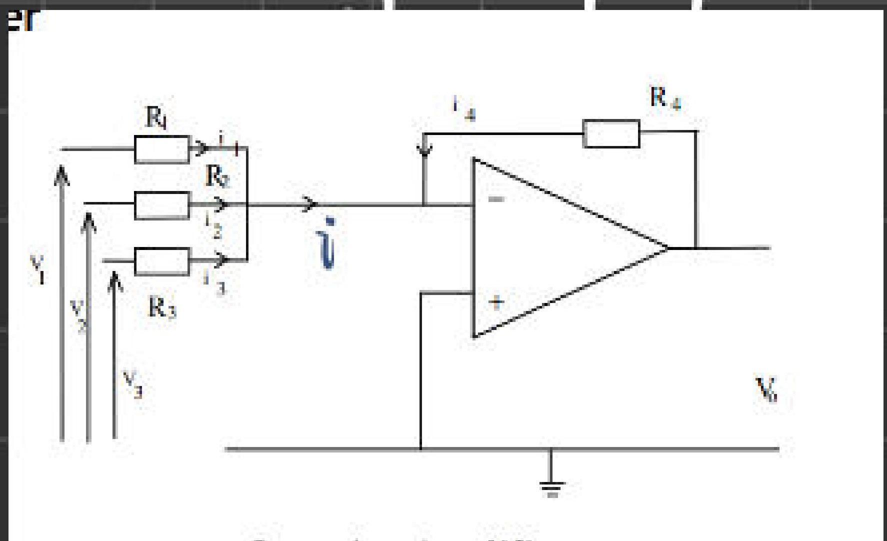


Unity Gain Amplifier

$$\left.\begin{aligned}V_o&=Av_i\\ v_i&=V_i-V_o\end{aligned}\right|\quad V_o=A(V_i-V_o)\Rightarrow\frac{V_o}{V_i}=\frac{A}{1+A}$$

$$A\to\infty\Rightarrow\frac{V_o}{V_i}=1$$

Summing Amplifier



Summing Amplifier

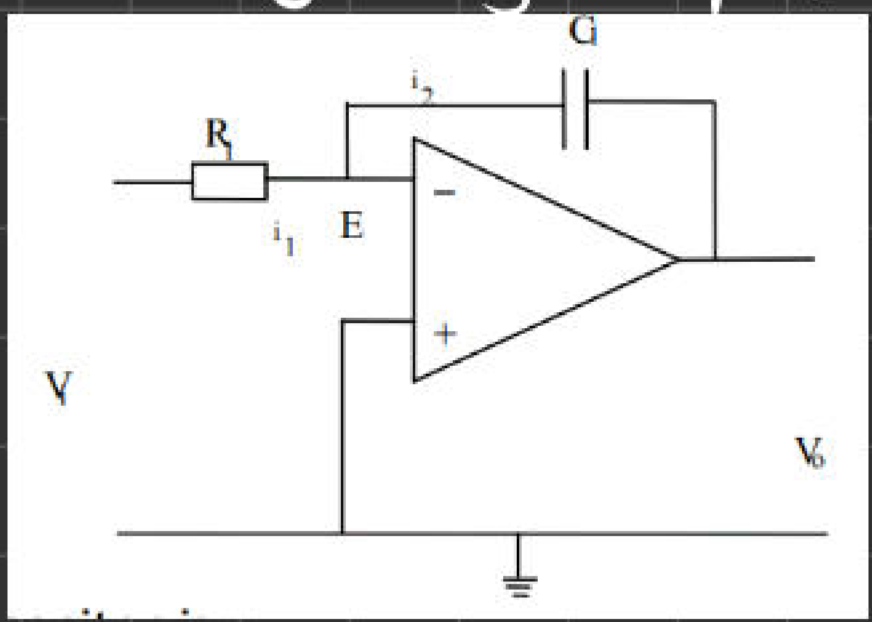
$$\left.\begin{aligned} i &= -i_4 \\ i &= i_1 + i_2 + i_3 \\ i_1 &= \frac{V_1}{R_1} \\ i_2 &= \frac{V_2}{R_2} \\ i_3 &= \frac{V_3}{R_3} \\ i_4 &= \frac{V_o}{R_4} \end{aligned}\right\} \Rightarrow \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = -\frac{V_o}{R_4}$$

$$\Rightarrow V_o = -R_4\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$$

when $R_4 = R_1 = R_2 = R_3$

$$V_o = -(V_1 + V_2 + V_3)$$

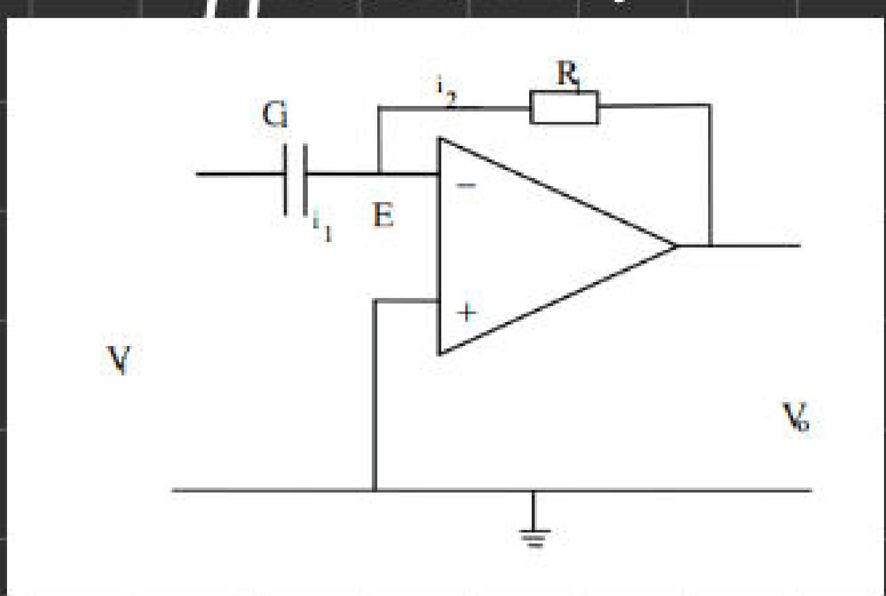
Integrating Amplifer



$$\left.\begin{aligned} i_1 &= -i_2 \\ i_1 &= \frac{V_1}{R_1} \\ i_2 &= C\frac{dV_o}{dt} \end{aligned}\right| \Rightarrow \frac{V_1}{R_1} = -C\frac{dV_o}{dt} \Rightarrow \frac{dV_o}{dt} = -\frac{V_i}{R_1 C}$$

$$\Rightarrow V_o = -\int_0^t \frac{V_i}{R_1 C}\,dt = -\frac{1}{R_1 C}\int_0^t V_i$$

Differentiators



$$\left.\begin{aligned} i_1 &= -i_2 \\ i_1 &= C\frac{dv_i}{dt} \\ i_2 &= \frac{V_o}{R_1} \end{aligned}\right| \Rightarrow C\frac{dv_i}{dt} = -\frac{V_o}{R_1} \Rightarrow V_o = -CR_1\frac{dv_i}{dt}$$

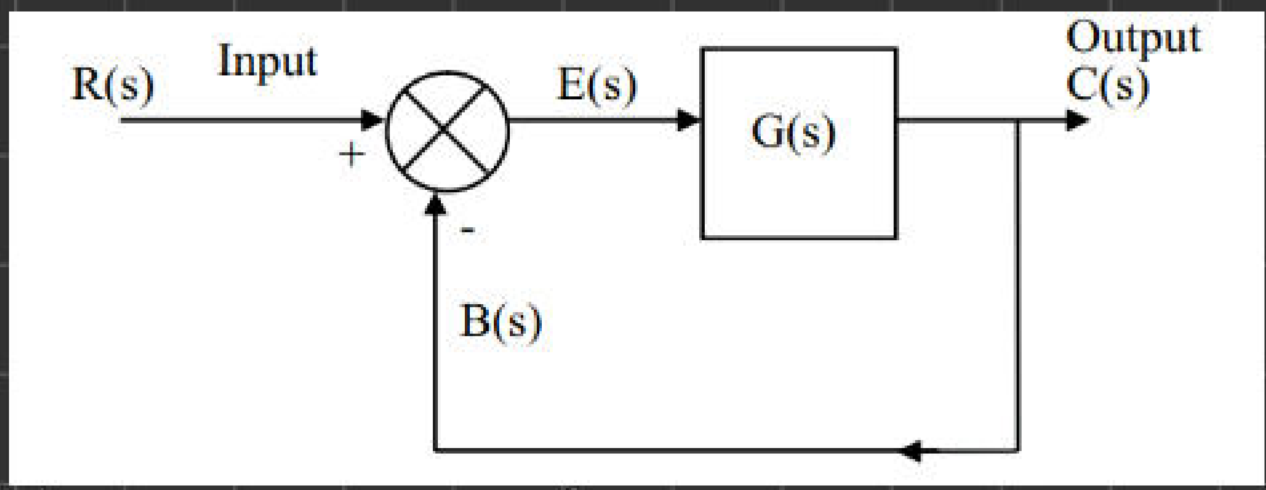
Type No and Final value and Steady State Error

Steady state error: $E(s) = \dfrac{R(s)}{1+G(s)H(s)}$ ← 输入

Final value theorem: $\lim\limits_{t\to\infty} f(t) = \lim\limits_{s\to 0} sF(s)$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$$

对于单位负反馈.



$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

for step input: $R(s) = \frac{K}{s}$ $\quad e_{ss} = \lim\limits_{s\to 0} s\times\frac{1}{s}\times\frac{K}{1+G(s)} = \lim\limits_{s\to 0}\frac{K}{1+G(s)} = \dfrac{K}{1+\lim\limits_{s\to 0} G(s)}$

令 $K_p = \lim\limits_{s\to 0} G(s)$ $\quad e_{ss} = \dfrac{K}{1+K_p}$ $\quad K_p$: positional error constant

for ramp input: $R(s) = \frac{K}{s^2}$ $\quad e_{ss} = \lim\limits_{s\to 0} s\times\frac{1}{s^2}\times\frac{K}{1+G(s)} = \lim\limits_{s\to 0}\frac{K}{s+sG(s)} = \dfrac{K}{\lim\limits_{s\to 0} sG(s)}$

令 $K_v = \lim\limits_{s\to 0} sG(s)$ $\quad e_{ss} = \dfrac{K}{K_v}$ $\quad K_v$: velocity error constant

for acceleration input: $R(s) = \frac{K}{2s^3}$ $\quad e_{ss} = \lim\limits_{s\to 0} s\times\frac{K}{2s^3}\times\frac{K}{1+G(s)} = \dfrac{K/2}{\lim\limits_{s\to 0} s^2G(s)}$

令 $K_a = \lim\limits_{s\to 0} s^2G(s)$ $\quad e_{ss} = \dfrac{K}{K_a}$ $\quad K_a$: acceleration error constant

$$G(s) = \frac{(s-z_1)(s-z_2)\cdots}{s^{\alpha}(s-p_1)(s-p_2)\cdots}$$

	$\alpha = 0$	$\alpha = 1$	$\alpha = 2$
for step input:	$K_p = K_p$, $e_{ss} = \frac{K}{1+K_p}$	$K_p = \infty$, $e_{ss} = 0$	$K_p = \infty$, $e_{ss} = 0$
for ramp input:	$K_v = 0$, $e_{ss} = \infty$	$K_v = K_v$, $e_{ss} = \frac{K}{K_v}$	$K_v = \infty$, $e_{ss} = 0$
for acc input:	$K_a = 0$, $e_{ss} = \infty$	$K_a = 0$, $e_{ss} = \infty$	$K_a = K_a$, $e_{ss} = \frac{K/2}{K_a}$

When there is a feedback block $\boxed{H}$, must use a different approach to calculate the error: $E(s) = [1 - T(s)]\times R(s)$

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s[1-T(s)]\times R(s)$$

Proportional Control: the output from the controller is proportional to the error input

proportional controller output(t) = $k_p \times e(t)$

$\Rightarrow \quad M(s) = k_p Z(s)$

PI control: To eliminate the steady state error, Integral control is added to the proportional control output.

Integral controller output(t) = $k_i \int e(t)dt$

$\Rightarrow \quad M(s) = k_i \frac{Z(s)}{s} \qquad (k_i = \frac{K}{T_i})$

PD control: the stability of a system can be improved and any tendency to overshoot can be reduced by adding derivate action.

Derivative controller output(t) = $k_d \frac{de(t)}{dt}$

$\Rightarrow \quad M(s) = k_d s Z(s) \qquad (k_d = K \cdot T_d)$

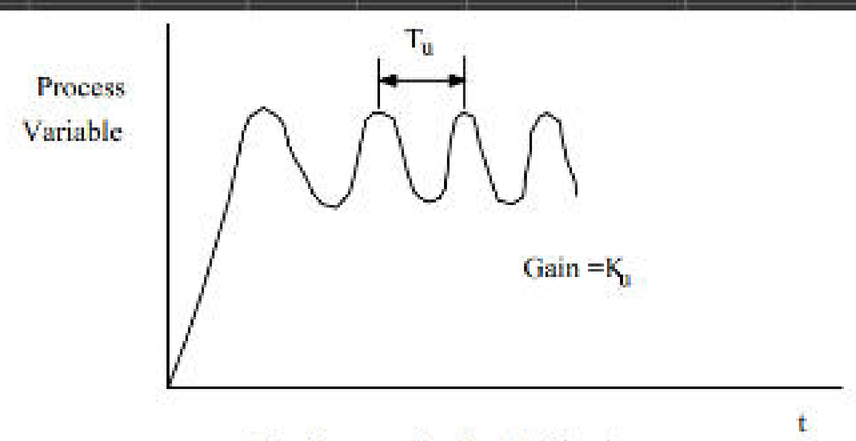
Proportional control: speed of responce

Integral action: accuracy of the final steady state value

derivative action: stability

PID controller Settings

1. closed loop 'Continuous cycling method'



Continuous Cycling Method

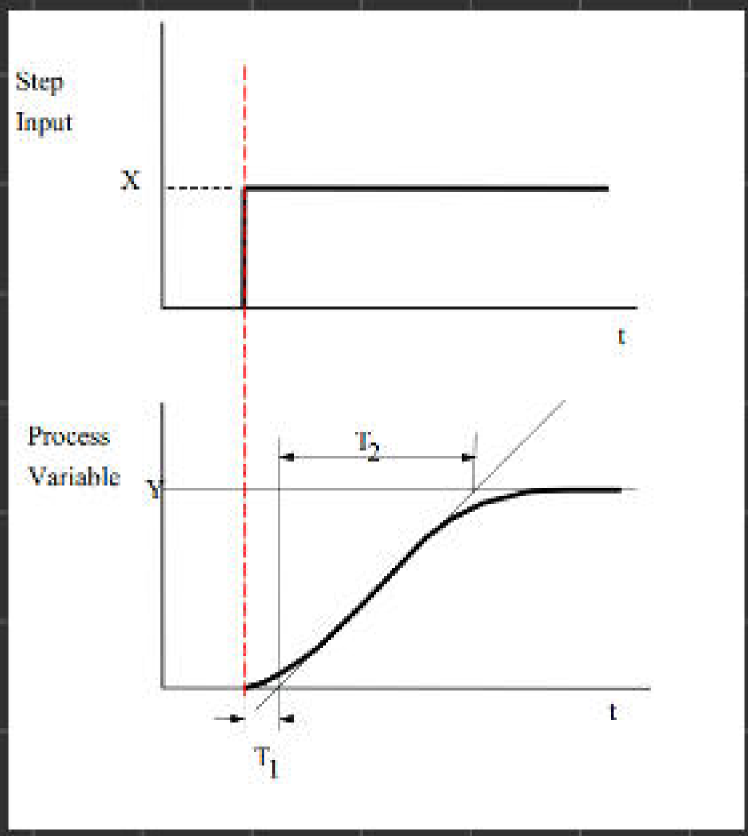
Control Action	K	Ti	Td
P	$0.5K_u$		
P+I	$0.45K_u$	$\frac{T_u}{1.2}$	
P+I+D	$0.6K_u$	$\frac{T_u}{2}$	$\frac{T_u}{8}$

For manual tuning process, the controller settings should be set to a minimum. The integral and derivative terms are elimated by setting $T_d = 0$ and $T_i = \infty$.

$\hookrightarrow (k_d = 0, k_i = 0)$

Adjust the system to the desired setpoint value.
Set the proportional gain to $k_p = 1$
After the output stabilizes at the set point value, a disturbance could be introduced to the system.
This will cause an oscillation in output.
we should obtain a sustained osillation, so we should increase the gain until the system reached sustained oscillation.

2. Open Loop 'Reaction Curve'



Control Action	K (Proportional Gain)	Ti (Integral Time)	Td (Derivative Time)
P	$\frac{T_2}{KT_1}$		
P+I	$0.9\frac{T_2}{KT_1}$	$\frac{T_1}{0.3}$	
P+I+D	$1.2\frac{T_2}{KT_1}$	$2T_1$	$0.5T_1$

T_1: delay / dead time

$K = \frac{Y}{X}$ — $Y \rightarrow \Delta Y$ change in output; $X \rightarrow \Delta X$ change in input.

manual → controller setting minimum → setpoint value

→ when stable, small step change → S shaped response curve

→ parameters are derived from the S curve